# Network Equivalence

## New Tools for Understanding Network Capacities

**by Michelle Effros**

**Caltech's Lee Center has played a critical role in my group's research into the field of network information theory.** Research in this area aims to understand the limits of network communication systems. One of the earliest results in the field is by Slepian and Wolf (1973); and considers a network with two transmitters describing information to a single receiver. For example, imagine two students, Alice and Bob, sitting in the same classroom. Both are taking notes, and each plans to send those notes to their friend Charlie, who is home sick with the flu. Since Alice and Bob are listening to the same lecture, their notes are likely to be similar but not identical. Each understands the material a little differently. Each misses a different portion of what is said in class. Slepian and Wolf set out to understand how long the descriptions sent to Charlie must be in order for him to learn everything contained in both sets of notes. If neither Alice nor Bob knew of the other's notes, then the total description length would be long; anything that appeared in both sets of notes would be described twice. In contrast, if Alice and Bob got together and prepared a single set of unified notes, the combined description could be much shorter. Slepian and Wolf studied the description length needed when Alice and Bob work independently. In this case, each knows that the other is also sending notes to Charlie, but neither knows what the other's notes say. Slepian and Wolf demonstrated that it is possible for Alice and Bob to independently describe their notes in a manner that allows Charlie to learn everything in both sets of notes from a pair of descriptions whose total length is no longer than the shortest description that Alice and Bob could have devised had they worked together. Thus Alice and Bob can do as well as if they worked together even though neither sees the other's notes (see Figure 1).

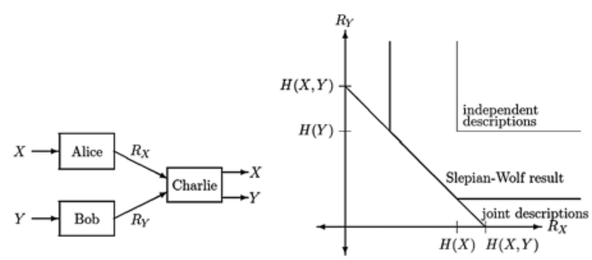


Figure 1: As shown by Slepian and Wolf, the shortest possible descriptions from Alice and Bob are far shorter than those required if neither knew the other were sending a description.

Slepian and Wolf's startling result illustrates both the power and the challenges associated with the field of network information theory. The power of the result lies both in the remarkable new understanding that it gives us about the world and in the technological advances that it enables. For example, more and more sensors are making their way into our environment. They monitor ground movement for signs of earthquakes and buildings for evidence of structural weakness; they track the movements of storms and their impact on tidal wave patterns; they help us to understand the health of our forests and oceans and air. As the number of sensors increases, the quantities of information to be gathered and the savings to be obtained by taking advantage of correlation between measurements increase as well. Advances from network information theory can help us to tackle these problems. On the other hand, the challenge illustrated by the Slepian-Wolf example is that the specific network studied is small because deriving this type of result is very difficult. In fact, almost four decades later, the gap between the complexity of the networks for which we have such complete characterizations and the complexity of the networks through which we communicate daily is growing ever larger.

With support from the Lee Center, we are pursuing methods for moving network information theory beyond such small networks. The goal is to develop rigorous, systematic tools for understanding the limits of large, complex communication networks. We wish to understand how much information these networks can carry, how to operate networks in order to enable the maximal reliable transfer of information, and how to design new networks in order to increase the amounts of information they can carry while decreasing the power and other resources required to meet those communication goals.

There are many challenges. First is the complexity of the networks themselves. In networks that rely on wireless components like mobile phones and sensors, information is transmitted through the air. Signals sent through this shared medium are overheard—to some extent—by all devices in the region. As a result, the signal that I receive on my cell phone is a very complicated mixture of the transmissions of all devices in range—even the devices with which I am not trying to communicate. The more devices communicate across a given network, the more complicated it becomes to understand how to protect information from noise and interference and to analyze how much information can be reliably delivered through the network.

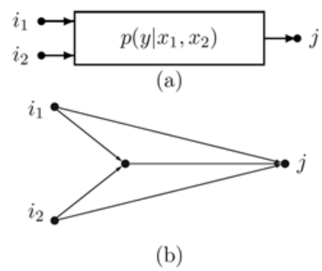

Figure 2: (a) A multiple access channel and (b) its noiseless bit-pipe model.

We have introduced a new equivalence theory for network capacity as a first step towards the development of systematic tools for network analysis. The idea is to model the behavior of wireless and noisy wireline network components in terms of noiseless wireline models. The resulting deterministic networks are easier to analyze, and a variety of tools for performing such analyses are already available. While the idea of breaking networks into simpler components and analyzing those components individually is not new, prior component models capture the performance of network components in isolation but fail to capture their full range of performance when these components are employed in larger network systems. Therefore, the key to our approach is to develop models for which we can demonstrate that no matter how the given component is used in a larger network context, the model can emulate the component's behavior. By building a library of wireless and noisy wireline component models, we can bound the communication capabilities of any network comprised of these components by the communication capabilities of a corresponding network of noiseless network components.

Examples of network components that we have modeled to date are the noisy wireline, broadcast, multiple access, and interference channels. The broadcast channel is a network component in which a single transmitter sends information to multiple receivers. The message intended for each receiver is different, but all receivers hear different noisy versions of everything sent by the transmitter. The multiple access channel is a network component that models interference (see Figure 2) wherein, multiple transmitters independently send information to a single receiver. What the receiver hears is some noisy combination of the signals sent by all transmitters. The interference channel combines the broadcast and interference effects into a single network component. In this component, each transmitter wishes to communicate with a single, distinct receiver. Since all transmitters send their signals simultaneously, each receiver hears a combination of its desired signal mixed with the signals intended for other receivers and with noise.

As this work continues, we are increasing the library of models and working to automate the process of separating complex practical networks into the components that most accurately capture network behavior as a whole. Combining the resulting algorithms with existing tools will enable analysis of increasingly complex communication networks. By understanding the limits of these network models and the strategies that achieve those limits, we hope to improve the use of current communication networks and create improved network designs in the future.

*Michelle Effros is Professor of Electrical Engineering.*

**Read more at:**
http://www.ee2.caltech.edu/people/faculty/effros.html

## References

[1] D. Slepian and J.K. Wolf, "Noiseless coding of correlated information sources," *IEEE Trans. Inf. Theory,* IT-**19**: 471-480, 1973.

[2] R. Koetter, M. Effros, and M. Médard, "On a Theory of Network Equivalence," In proceedings of the IEEE Information Theory Workshop, Volos, Greece, June 2009.

[3] R. Koetter, M. Effros, and M. Médard, "Beyond Network Equivalence," In *Allerton Annual Conference on Communications, Control, and Computing*, Monticello, IL, September 2009.

[4] M. Effros, "On capacity upper bounds for a simple family of wireless networks," In proceedings of the Information Theory and Applications Workshop, San Diego, CA, February 2010.