

FINDING THE BEST QoS PATH IN A GILBERT CHANNEL NETWORK

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Abstract—Many different types of modern wired and wireless communication links can be mathematically described as discrete-time Gilbert channels. In this extended abstract, we present an exact method of calculating the best path in a network of discrete-time Gilbert channels, each of which is defined as a Markov chain with two states. In the “Good” state of the chain, the channel produces no erasure, and in the “Bad” state of the chain, the channel produces an erasure. Our method relies on a modified version of the Dijkstra’s algorithm, which we customize to operate on sets of Gilbert channel parameters, instead of real numbers. We prove that the Gilbert channels obeys a certain set of algebraic properties which makes it compatible with our algorithm.

I. INTRODUCTION

To the first approximation, many different types of modern wired and wireless communication links can be considered memoryless. Thus, depending on the types of signal used, these links can be mathematically modeled as binary erasure channels, binary symmetric channels, or AWGN channels, etc. However, in many applications, accurate behavior and performance analysis has to take into account the fact that in reality, channel distortion exhibits memory.

The Gilbert channel model [1] is one of the simplest models for channels with memory. Mathematically, it is nothing more than a Markov chain with two states: G and B , with their outgoing transition probability values denoted by p and q , respectively. In this extended abstract, we assign B to be the state where undesirable events occur, e.g., packet loss, packet error, fading higher than a predetermined threshold, etc.

Despite (also because of) its apparent simplicity, the Gilbert Channel Model (GCM) has been widely used in analyzing a wide variety of networks [2]. For example, GCM is used to analyze the performance of slotted ALOHA over fading communications channels [3], or correlated loss over TCP/IP networks [4]. Other relevant examples include performance analysis of real-time wireless communications [5]. For many wireless fading channels, GCM is a very attractive alternative to sophisticated models such as Hidden Markov models (HMMs) [6]. For one, GCMs are analytically tractable. In addition, it can easily produce packet-level network QoS.

GCM becomes even more important with the advent of Universal Mobile Telecommunications Systems (UMTS) networks where multimedia (especially speech) and data packets will have to coexist in the underlying common Wideband Code-Division Multiple Access (WCDMA) networks.

The recent UMTS recommendations for QoS measures is based on a set of user satisfaction assessments of individual speech and data sessions [7]. For speech services, user satisfaction drops significantly in the presence of long spans (in terms of packets) of service outage. For data services, disruption and termination comes from successive retransmissions.

These new QoS measures require the next level of approximation of behavioral analysis that is not available from system outage probability analysis (which inherently assumes zero correlation between the outages) commonly found in CDMA literature [8], [9]. Recently, detailed analysis (in the context of QoS and capacity) of WCDMA [10], [11] incorporates correlated outage behavior by using GCM.

This extended abstract attempts to outline our method of finding the best QoS path in networks where GCM can be applied, and wherever shortest path routing is used. In particular, it also applies to the WCDMA case that we just discussed.

II. GILBERT CHANNEL NETWORK

As previously mentioned, a discrete-time Gilbert Channel (GC) is defined as a Markov chain with two states, G and B , whose outgoing transition probability values are denoted by p and q , respectively, with $0 \leq p, q \leq 1$.

The Markov chain produces a deterministic binary output value that indicates whether or not the channel produces an erasure event. In the G (or “Good”) state, the output is a 0, which indicates no erasure, whereas in the B (or “Bad”) state, the output is a 1, which indicates a channel erasure. Mathematically, we can represent the space of all GC as the space $\Lambda = [0, 1] \times [0, 1]$, and a particular GC instance by a point $\lambda = (p, q) \in \Lambda$. In particular, when $p = 0$ or $q = 0$, the GC converges to a deterministic steady-state behavior.

Suppose we represent all networks by the standard $\mathcal{G} = (V, E)$ node- and edge-sets. Then a Gilbert Channel Network (GCN) is just \mathcal{G} with the additional specification that its edge weights are taken from Λ . This brings out several important questions about GCNs. First, is it possible to combine GCN edges into a path, and if so, is it possible to derive a combined path weight from its edge weights? Second, is it possible to compare path weights to find the best path in a GCN? Third, can we cast this problem as a generalized shortest path problem that can be solved using the Generalized Dijkstra’s Algorithm (GDA) [12]? If so, we say that the GCN is compatible with the GDA, and the best path can be found in $O(V^2)$.

As always, we obtain the most definitive answer by paraphrasing the questions into its equivalent formal mathematical questions. If we define an algebra \mathbf{B} by combining Λ and two operators: \oplus and \preceq to represent the general operations of (1) adding two edge weights together (to ultimately construct a path), and (2) comparing two weight values, the questions is identical to proving the following proposition:

Proposition 1. *An algebra $\mathbf{B} = (\Lambda, \oplus)$ and a total order \preceq is compatible with the GDA if and only if it satisfies all the properties in the set denoted by \mathbf{P} below:*

P1 *is a commutative monoid, that is, for $a, b, c \in \Lambda$:*

- Λ is closed under \oplus : $a \oplus b \in \Lambda$;
- \oplus is associative : $a \oplus (b \oplus c) = (a \oplus b) \oplus c$;
- 0 is the identity : $a \oplus 0 = 0 \oplus a = a$;
- \oplus is commutative : $a \oplus b = b \oplus a$.

P2 *There exists $\infty \in \Lambda$ | $a \oplus \infty = \infty \oplus a = \infty$.*

P3 *\preceq is a total order on Λ , i.e., \preceq is :*

- reflexive: $a \preceq a$;
- anti-symmetric: *if $a \preceq b$ and $b \preceq a$ then $a = b$;*
- transitive: *if $a \preceq b$ and $b \preceq c$ then $a \preceq c$;*
- total: *for every $a, b \in \Lambda$ either $a \preceq b$ or $b \preceq a$.*

P4 *There exists the least element 0 that satisfies $0 \preceq a$.*

P5 *$a \oplus c \prec b \oplus c$ if $a \prec b$ and $c \in \Lambda - \{\infty\}$.*

The Generalized Dijkstra's Algorithm (GDA) below [12] is a modified version of Dijkstra's Algorithm that can operate on any metric of choice as long as the metric is compatible. In this extended abstract we attempt to provide a sketch the proofs that \mathbf{B} is compatible with the GDA, deferring the complete proof to the full paper version of this abstract.

In the following listing, \mathcal{G} is the GCN, while s is the source node of \mathcal{G} . The functions β and N return (1) the edge weight connecting its two argument nodes, and (2) the neighbors of its argument, respectively. The arrays l and π keep track of cumulative path length and predecessor node, respectively.

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1: procedure GDA( $\mathcal{G}, \beta, s$ )
2:   for all  $v \in V$  do
3:      $l[v] \leftarrow \infty$ 
4:      $\pi[v] \leftarrow \text{NIL}$ 
5:    $Q \leftarrow V$ 
6:    $l[s] \leftarrow 0$ 
7:   while  $Q \neq \emptyset$  do
8:      $u \leftarrow \text{MIN}(Q)$ ;  $Q \leftarrow Q \setminus u$ 
9:     for all node  $v \in N(u)$  do
10:      if  $l[v] \succ l[u] \oplus \beta(u, v)$  then
11:         $l[v] \leftarrow l[u] \oplus \beta(u, v)$ 
12:         $\pi[v] \leftarrow u$ 

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Interested readers can consult [13] for a description of the standard Dijkstra's Algorithm, which is practically identical to the GDA except for the relaxation step, where the \oplus and \preceq operators act on a generalized metric space Λ (instead of the equivalent step in DA, where $+$ and \leq operators act on \mathbb{R}). We shall shortly define \preceq for our channel.

In our previous work, we presented methods based on the GDA to find the best QoS path in networks containing (exclusively of) Binary Erasure Channels (BEC) [14], Binary Symmetric Channels (BSC) [15], and with a very high probability, the Additive White Gaussian Noise (AWGN) channels.

Since the GC is a generalization of the BEC (a BEC with loss parameter p is just a GC with $q = 1 - p$), the results presented here extend our previous results for the BEC network by introducing a simple Markovian memory.

Finally, before we begin presenting the sketch of the proof, first we have to precisely define in the next section what the operators \oplus and \preceq really mean, and define more precisely the various symbols for our network problem.

III. NETWORK MODEL

The Gilbert Channel Network (GCN) is modeled as a digraph $G = (V, E)$, where V , E , and Π are the node, edge and path sets of G . Two nodes s and $d \in V$ are the source and destination nodes, and $\Pi \subset \Pi$ denotes the set of all paths from s to d .

What makes the GCN unique is the space of its edge weight values $\lambda \in \Lambda$ that contains all pairs of parameter values (p, q) , whose components represent the transition probability p from the desirable state G to the undesirable state B , and vice versa for q . In this definition, the probability values of staying in the same states are $1 - p$ and $1 - q$, respectively. Each edge $e_i \in E$ has its own edge weight $\lambda_i \in \Lambda$. The special value $\lambda = \infty$ denotes the absence of connection between two nodes.

A path $\pi \in \Pi$ whose nodes $V_\pi \subset V$ are connected by $E_\pi \subset E$ is denoted by either $\langle v_0, \dots, v_J \rangle$ or $\langle e_1, \dots, e_J \rangle$. The symbol $\langle v_i, v_{i+1} \rangle$ denotes the edge (path) connecting the two (non-) adjacent nodes v_i and v_{i+1} in a network of GC's.

Define the function $X : \Lambda \rightarrow \mathcal{M}$ that measures a user-defined degree of reliability associated with a path or an edge. For convenience, let us define the function $\beta : \Pi \rightarrow \Lambda$ that maps a path π (or an edge e_i) into a parameter λ_π (or λ_i), and a shorthand notation $X(e) = X(\beta(e))$.

Let us assume that the addition operation is defined in Λ and \mathcal{M} and is denoted by \oplus . If $x_1 = X(e_1)$, $x_2 = X(e_2)$, $\lambda_1 = \beta(e_1)$, $\lambda_2 = \beta(e_2)$, and $\pi = \langle e_1, e_2 \rangle$, then we say $x_\pi = x_1 \oplus x_2$ and $\lambda_\pi = \lambda_1 \oplus \lambda_2$. For these expressions to make sense, the \oplus , Λ and \mathcal{M} have to obey the algebraic properties \mathbf{P} that we discussed in the previous section. Having defined \oplus , we can now compute x_π and λ_π in terms of x_i and λ_i using a generalized summation: $x_\pi = \bigoplus x_i$ and $\lambda_\pi = \bigoplus \lambda_i$.

We call the pairs (Λ, \oplus) and (\mathcal{M}, \oplus) the \mathbf{B} and \mathbf{X} algebras, with their associated β and X functions, respectively. To calculate the optimal network path π^* , however, we need more than \oplus , β and X to evaluate expressions like these:

$$\begin{aligned}
 x^* &= \min_{\pi} \{ x_\pi \mid \pi \in \Pi \} \\
 \lambda^* &= \min_{\pi} \{ \lambda_\pi \mid \pi \in \Pi \}
 \end{aligned} \tag{1}$$

We need a total order operator \preceq acting on Λ and \mathcal{M} to evaluate expressions such as $x_\pi \preceq x_{\pi'}$ and $\lambda_\pi \preceq \lambda_{\pi'}$. Once the operator \preceq is defined, then the above expression makes sense.

For simplicity, the metrics induced from the coupling of \preceq with \mathbf{B} and \mathbf{X} are also referred to as the \mathbf{B} and \mathbf{X} metrics, respectively. With reference to the source node s and the destination node d , the optimal path π^* is the path with the “shortest” length when measured in the \mathbf{B} and \mathbf{X} metrics, both of which use the \preceq operator that is defined as follows:

Definition 2. $\lambda_1 \preceq \lambda_2$ iff $X(\lambda_1) \preceq X(\lambda_2)$.

Before defining the \oplus operator for \mathbf{B} and \mathbf{X} , let us define the $X(e)$ function in this paper as the function that measures the long term probability of channel erasures of an edge e . This definition of $X(e)$ is compatible to the analysis in our previous work [14] of the standard memoryless erasure channel, which is a special case of the GC with $q = 1 - p$.

Denote by $S_i(t)$ the state at a particular (discrete) time t of the Markov chain associated with the GC on an edge e_i , and by $F_i(t)$ the corresponding Markov chain output. For brevity, the dependence on t is omitted from the notation when it is obvious from the context. For the memoryless case, the probability for observing the failure indicator F_i is given by:

$$P(F_i = f_i) = \begin{cases} 1 - p & , f_i = 0 \text{ (no failure)} \\ p & , f_i = 1 \text{ (failure)}. \end{cases} \quad (2)$$

The \mathbf{B} algebra is such that any two adjacent edges e_1 and e_2 with parameters λ_1 and λ_2 (and their corresponding long term erasure probabilities x_1 and x_2) can be viewed as a single edge with parameter $\lambda = \lambda_1 \oplus \lambda_2 = p_1 \oplus p_2$ defined by:

$$\begin{aligned} p_1 \oplus p_2 &= 1 - (1 - p_1)(1 - p_2) \\ x_1 \oplus x_2 &= p_1 \oplus p_2 \end{aligned} \quad (3)$$

In the memoryless case, the scalar parameter that plays the role of λ is p , and from the law of large numbers, we know that the value of $X(\lambda)$ is identical to the value of probability function P above. Therefore, in this case, the \preceq operator is defined by the following relation: $\lambda_1 \preceq \lambda_2$ iff $p_1 \preceq p_2$.

To model channel erasures with memory, we return to our assumption that at each time t , the state $S_i(t)$ of an edge e_i could either be Good (labeled G) or Bad (labeled B), and that $S_i(t)$ is Markovian. In the good state, the output $F_i(t) = 0$ (no erasure), and in the bad state, the output $F_i(t) = 1$ (erasure).

As is true with any two-state Markov Chain, a GC is completely defined by its transition probabilities. Let us denote the transition probability from the good to the bad state by $p = P(S_i(t) = B \mid S_i(t-1) = G)$, and from the bad to the good state by $q = P(S_i(t) = G \mid S_i(t-1) = B)$.

In a GCN, each edge e_i is given an edge weight λ_i that is completely characterized by the vector (p_i, q_i) . As before, the \mathbf{B} algebra is defined in such a way that any two adjacent edges e_1 and e_2 with parameters λ_1 and λ_2 can be viewed as a single edge with parameter $\lambda = \lambda_1 \oplus \lambda_2 = p_1 \oplus p_2$.

As expected, the definition of the \oplus operator for the GCs is a little bit more complex than the definition of the \oplus operator for the memoryless case. To define the \oplus for GCs, we have to take an indirect approach that uses the definition of the X function and its associated \mathbf{X} algebra.

Like in the case of memoryless erasure channel, the function X measures the long term erasure probability. The difference is that, for erasure channels with memory, Markov chains are involved and the function X actually measures the stationary probability of finding the edge e_i in the bad state, i.e.: $X_B(e_i) = P(S_i = B)$ (when the edge in question is not ambiguous, we simply refer to this quantity as X_B). This probability is given by the following expression:

$$X_s(e_i) = P(S_i = s) = \begin{cases} \frac{q_i}{p_i + q_i} & , s = G \\ \frac{p_i}{p_i + q_i} & , s = B. \end{cases} \quad (4)$$

As expected, $X_G + X_B = 1$, which reflects the fact that for any observation, the edge e_i has to be in one of the states.

Continuing our effort to define \oplus , we then observe that since the failure events for two adjacent edges e_1 and e_2 are independent of one another, the stationary probability of finding $e = \langle e_1, e_2 \rangle$ in the good state is simply the product of $X_G(e_1)$ and $X_G(e_2)$. Switching to the \mathbf{X} algebra, we claim that:

$$\begin{aligned} X_G(e) &= X_G(e_1) \oplus X_G(e_2) \\ &= X_G(e_1) X_G(e_2) \\ &= \frac{q_1}{p_1 + q_1} \frac{q_2}{p_2 + q_2} \\ &= \frac{q_1 q_2}{(p_1 + q_1)(p_2 + q_2)} \\ &= \frac{q}{p + q} \end{aligned} \quad (5)$$

Equation (5) gives one equation for solving the two unknowns p and q , which are the transition probabilities of the combined edge e . The other equation is from the definition of p :

$$p = 1 - (1 - p_1)(1 - p_2) \quad (6)$$

The above equation mathematically states that for the edge e to be in the Good state, both e_1 and e_2 have to be in their respective Good states. If either one of them makes a transition to the Bad state, then e will also transition to its Bad state.

Lemma 3. Let $\lambda_1 = (p_1, q_1)$ and $\lambda_2 = (p_2, q_2)$. Suppose $\lambda = (p, q) = \lambda_1 \oplus \lambda_2$. If we denote by $p = p(p_1, q_1, p_2, q_2)$ and $q = q(p_1, q_1, p_2, q_2)$, then:

$$\begin{aligned} p &= 1 - (1 - p_1)(1 - p_2) \\ q &= p \cdot \frac{q_1 q_2}{(p_1 + q_1)(p_2 + q_2) - q_1 q_2} \end{aligned} \quad (7)$$

PROOF: We have already discussed the first equation and will not repeat the proof. The proof for q is as follows. From the last equation in (5), we first solve for q ,

$$q = p \cdot \frac{X_G(e)}{1 - X_G(e)} \quad (8)$$

$$X_G(e) = \frac{q_1 q_2}{(p_1 + q_1)(p_2 + q_2)} \quad (9)$$

and substitute the full expression of $X_G(e)$. Following some algebraic simplification, q can be easily obtained. \square

Finally, to complete our discussion on the algebraic operators for GCs, we provide the following definition for \preceq . If we let $\lambda_1 = \beta(e_1)$ and $\lambda_2 = \beta(e_2)$, then:

$$\lambda_1 \preceq \lambda_2 \quad \text{iff} \quad X_B(\lambda_1) \preceq X_B(\lambda_2). \quad (10)$$

Having defined \oplus and \preceq , we are now ready to show that GCs have **B** (and **X**) algebras that are compatible to the Generalized Dijkstra's Algorithm (GDA), a modified version of the original Dijkstra's Algorithm (DA), which has been proven to have a time-complexity of at most $O(V^2)$.

Theorem 4. The algebra **B** = (Λ, \oplus) in the memoriless case and its respective total order \preceq satisfy all the properties in **P**, and thus compatible with the GDA.

PROOF: We will prove the properties one by one. Recall that $\Lambda = [0, 1]$. Let the variables a , b , and c in **P** correspond to the parameters p_1 , p_2 , and p_3 (as defined in equation (3) of three consecutive edges e_1 , e_2 , and e_3).

P1 Except for closure, the monoid properties can be proven by performing an algebraic manipulation of equation (3). Recall that $\Lambda = [0, 1] \cap \infty$. Closure is obvious if $a, b \in [0, 1]$. In the proof for P2, we identify the element $\infty \in [0, 1]$. Thus, closure is guaranteed when $a = \infty$ or $b = \infty$.

This gives us one interpretation of ∞ ; i.e., it is an alias for an element in $[0, 1]$. We can also adapt an alternative interpretation of an edge weight of ∞ , using it as a notation to indicate the absence of any connection between two nodes. In this interpretation, $a \oplus b = \infty$, because if either $a = \beta(\langle v_1, v_2 \rangle) = \infty$ or $b = \beta(\langle v_2, v_3 \rangle) = \infty$, then $\beta(\langle v_1, v_3 \rangle) = \infty$.

P2 The proof is obvious if ∞ is a notation for the absence of connection (as we just discussed), or alternatively, $\infty = 1 \in [0, 1]$ also satisfies this property.

P3 The proof follows the properties of Λ , which is just \mathbb{R} .

P4 Same as above.

P5 The proof is obvious if $b = \infty$. However, if $b \neq \infty$, then by substituting a, b , and c into (3) we obtain the two values $a \oplus c$ and $b \oplus c \in [0, 1]$ given by:

$$a \oplus c = 1 - (1 - a)(1 - c)$$

$$b \oplus c = 1 - (1 - b)(1 - c).$$

Since both are in $[0, 1]$, the order \prec is just $<$, and the expression $a \oplus c \prec b \oplus c$ is equivalent to the inequality $(a \oplus c) - (b \oplus c) < 0$, which can be simplified into:

$$(1 - c)(a - b) < 0$$

If $c \in [1, 0] - \{\infty\}$ then $c \neq \infty$, but that means $c < 1$, or $(1 - c)(a - b) < 0$ (because $a \prec b$ and $b \neq \infty$). Thus, the above inequality is true, and we have proven **P**. \square

Theorem 5. The algebra **B** = (Λ, \oplus) for Gilbert Channels and its total order \preceq satisfy all the properties in **P**, and thus compatible with the GDA.

PROOF: The proof is similar to that of theorem 4.

P1 We need to prove that if $\Lambda = [0, 1] \times [0, 1]$ then $(p_1, q_1) \oplus (p_2, q_2) \in \Lambda$. First observe that $p(p_1, q_1, p_2, q_2) = p_1 \oplus p_2$. Recall that we have proven the closure property of this type of function in the previous theorem. Therefore, closure is satisfied iff $0 \leq q(p_1, q_1, p_2, q_2) \leq 1$. From (8)

$$q = p \cdot \frac{X_G(e)}{1 - X_G(e)} \leq 1 \quad (11)$$

Since $0 \leq X_G(e) \leq 1$, then $X_G(e)/(1 - X_G(e)) > 0$. In addition, if we assume that closure on q is satisfied, then we have $0 \leq q \leq 1$, and the following must be true:

$$q \leq 1$$

$$p \leq (1 - X_G(e)) / X_G(e)$$

$$p_1 + p_2 - p_1 p_2 \leq [(p_1 + q_1)(p_2 + q_2) - q_1 q_2] / (q_1 q_2)$$

$$p_1 q_1 q_2 + p_2 q_1 q_2 - p_1 p_2 q_1 q_2 \leq p_1 p_2 + p_1 q_2 + p_2 q_1$$

$$p_1 q_1 q_2 + p_2 q_1 q_2 \leq p_1 p_2 q_1 q_2 + p_1 p_2 + p_1 q_2 + p_2 q_1$$

$$p_1 q_1 q_2 + p_2 q_1 q_2 \leq p_1 q_2 + p_2 q_1$$

$$q_1(p_1 q_2) + q_2(p_2 q_1) \leq p_1 q_2 + p_2 q_1$$

In the above, we first derived the second inequality using (11). Next, both sides of the inequality are expanded using (7). We then multiply both sides with $q_1 q_2 \geq 0$ and add $p_1 p_2 q_1 q_2$ to both sides. At this point, we observe the inequality involves only positive terms. Removing two positive terms on the right hand side does not change the inequality. Finally, by appropriately factoring the terms on the left hand side, and using the fact that $q_1, q_2 \leq 1$, we reach the final inequality that is always true.

To prove associativity, we can use a symbolic algebraic manipulator to verify that the following is true:

$$\begin{aligned} & p(p(p_1, q_1, p_2, q_2), q(p_1, q_1, p_2, q_2), p_3, q_3) = \\ & p(p_1, q_1, p(p_2, q_2, p_3, q_3), q(p_2, q_2, p_3, q_3)) \\ & q(p(p_1, q_1, p_2, q_2), q(p_1, q_1, p_2, q_2), p_3, q_3) = \\ & q(p_1, q_1, p(p_2, q_2, p_3, q_3), q(p_2, q_2, p_3, q_3)) \end{aligned} \quad (12)$$

Likewise, commutativity can be proven by verifying:

$$\begin{aligned} & p(p_1, q_1, p_2, q_2) = p(p_2, q_2, p_1, q_1) \quad \text{and} \\ & q(p_1, q_1, p_2, q_2) = q(p_2, q_2, p_1, q_1) \end{aligned} \quad (13)$$

and finally, it is easy to prove that for any $q \in [0, 1]$, the element $(0, q)$ satisfies the property of the 0 element.

P2 The ∞ element is $(p, q) = (1, 0) \in [0, 1]$.

P3 Equation (8) defines q as a linear function of p with a slope of $X_G(e)/(1 - X_G(e))$. Thus, each value of $X_G(e)$ corresponds to a line $q(p)$ that intersects the origin $(p, q) = (0, 0)$. These lines are therefore contour lines for the different values of $X_G(e) \in [0, 1]$. The expression $(p_1, q_1) \preceq (p_2, q_2)$ then simply compares the slopes of the two lines (the steeper the slope, the better). P3 then follows from the ordering properties of \mathbb{R} .

P4 By P1, $(0, q)$ satisfies the criteria for the 0 element. If e is an edge such that $\beta(e) = (0, q)$, then we have $X_B(e) < X_B(f)$ for all edges f with $\beta(f) \neq (0, q')$.

P5 Suppose a , b , and c are the values of $\beta(e_1)$, $\beta(e_2)$, and $\beta(e_3)$, respectively. First, using the fact that $X_B = 1 - X_G$, and recalling that $X_G(\cdot), X_B(\cdot) \in \mathbb{R}$, we prove the isotonicity property as follows:

$$\begin{aligned}
a &< b \\
X_B(a) &\leq X_B(b) \\
X_G(a) &\geq X_G(b) \\
X_G(a)X_G(c) &\geq X_G(b)X_G(c) \\
X_G(a \oplus c) &\geq X_G(b \oplus c) \\
1 - X_G(a \oplus c) &\leq 1 - X_G(b \oplus c) \\
X_B(a \oplus c) &\leq X_B(b \oplus c) \\
a \oplus c &< b \oplus c
\end{aligned}$$

Note that just as in our definition for \oplus , in our proof for isotonicity, we made heavy use of the relationship between the \mathbf{B} and \mathbf{X} algebras and their \leq operators. \square

Theorem 5 shows that the GDA can be used to reliably compute the network path that has the minimum long term channel erasure probability, which is an important QoS metric for a variety of modern communication systems. Mathematically, we state this result in the following corollary:

Corollary 6. *The long term erasure probability $X(\lambda)$ is a non-decreasing function of its argument λ . Consequently, the path with minimum λ obtained from the GDA is also the path with the minimum long term erasure probability $X(\lambda)$.*

PROOF: The proof follows from our definition for \leq :

IV. CONCLUSION

In this extended abstract, we have presented a method to find the best QoS path in a network where each link is modeled as a Gilbert channel. We briefly discussed why the Gilbert Channel Model is of considerable importance in modern communication systems. First, among the different models that are used to analyze a wide variety of communication channels with memory, the Gilbert Channel Model is one of the simplest and the most mathematically tractable. More importantly, the model was recently proposed as a tool for analyzing various QoS aspects of the multi-service UMTS-over-WCDMA networks.

Building from our previous body of work in the area of best-QoS routing for discrete memoryless channel networks, we proposed to use the Generalized Dijkstra's Algorithm to a class of communication network that can be modeled as a network of Gilbert Channels. This important class of network includes the TCP/IP networks and many types of wireless fading channel networks with correlated errors. In this extended abstract, we rigorously proved the algebraic correctness of our approach.

We described the various properties that are required in order to guarantee that the GDA can be used on any Gilbert Channel Networks. We proved that all compatibility properties are satisfied, and thus proved that the GDA returns the loop-free path with the best QoS in a network of Gilbert channels.

Future work includes extending the result to the general Finite-State Markov Chain channels and general Discrete Memoryless Channels. Numerical experiments can also be conducted to verify the correctness of this algebraic approach. In these experiments, our algorithm can be tested on random graphs with topologies that closely resemble the Internet. To generate such graphs, specialized random graph algorithms [16] can be used in combination with an edge weight generator that uses an experimentally determined distribution.

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