Formal Verification of Distributed Algorithms

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Overview

Improve distributed software **reliability**

Ease **proof** burden for concrete algorithms

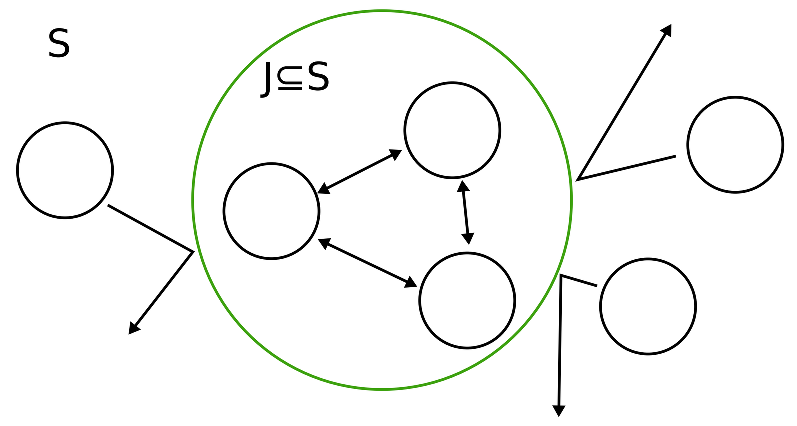
Stepwise **refinement** and program transformation

Organize **library** of theorems for distributed systems (PVS)

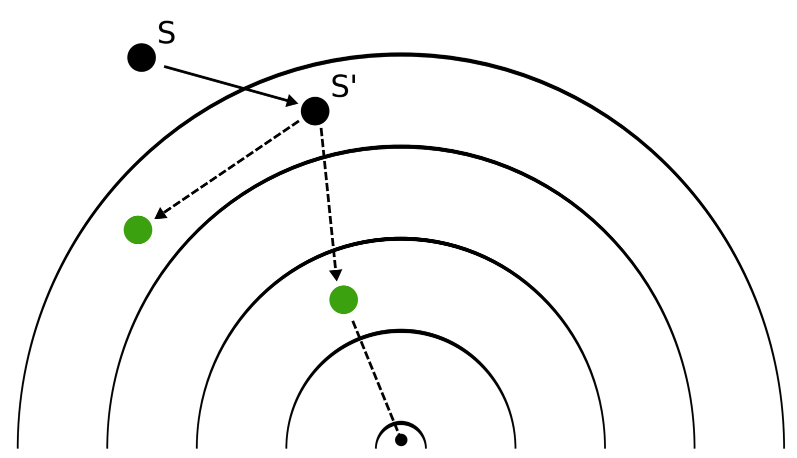
Transform theorem proven code into executable code (Java/Erlang)

Improve **teaching**

Local Interactions...



... Global Implications



Abstraction

Abstractions

Incorporate fundamental abstractions of distributed algorithms that are generic enough to be applicable to a large class of problems

Fundamental Functions

Binary operator	∘: T → T
Composition function	f: S, {K⊆S}, ∘ → T
Transition Predicate	t: S, S → Bool

Correctness Properties

Safety: invariant function with respect to start state

$$t(S^0, S) \Rightarrow f(S, \{K=S\}, \circ) = f(S^0, \{K=S\}, \circ)$$

Progress: variant function to a well founded set

$$\rho: T \rightarrow T \mid T \text{ is well founded}$$

Application: Consensus

Abstractions

Given state S with n agents, want final state to be function of start state S*: $\forall j \in S: S^*(j) = f(S^0)$

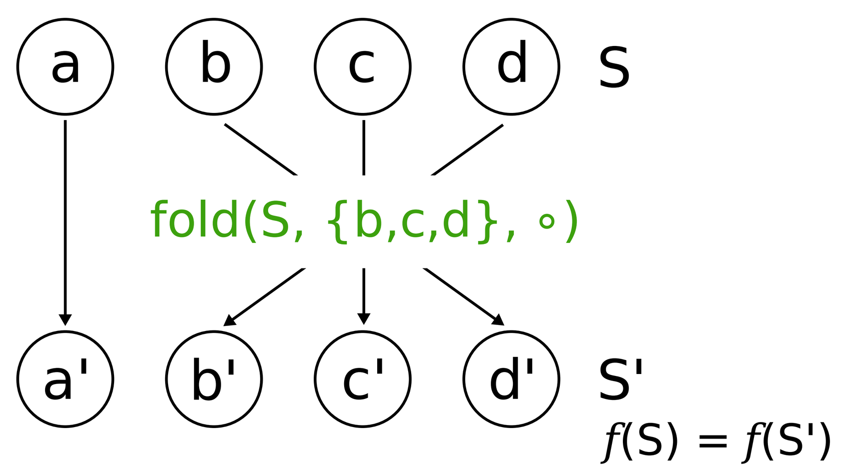
∘: commutative ∧ associative ∧ (idempotent ∨ super idempotent)

f is aggregation (fold) of composable binary operator ∘
$f(S) = S(0) \circ S(1) \circ \ldots \circ S(n)$

$$\equiv \text{fold}(S, \circ) = \begin{cases} S(0) \text{ if } |S| = 1 \\ \text{fold}(S, \{J \setminus S(n)\}, \circ) \circ S(n) \text{ otherwise} \end{cases}$$

Transition	$t(S, S') \equiv \forall j: \exists J: j \in J \wedge S'(j) = \text{fold}(S, J, \circ)$
Safety	$\forall S, S': t(S, S') \Rightarrow \text{fold}(S, J, \circ) = \text{fold}(S', J, \circ)$
Progress	$\lvert\{ a \text{ where } S(a) = \text{fold}(S^0, \{a\}, \circ) \}\rvert$

Visual Transition



Proof Refinement

∘(S, S) → T

fold(S, J, ∘) → T

commutative, associative

min/max — gcd/lcm (idempotent)

Average — Convex Hull (super idempotent)

Object Refinement

Operator

FoldableOperator

Min — GCD — ... — Cvx Hull

Application: Graphs

Abstractions

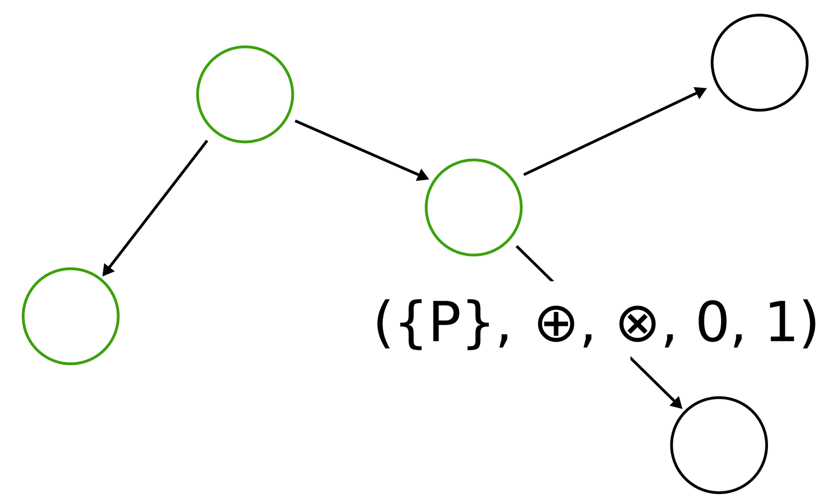
Given source graph G and root vertex R, produce target graph that solves given algebraic path problem

∘: semiring with additional constraints
idempotent ($x \oplus x = x$), and
partial order ≤ over {P}

f is a depth first graph traversal method

Transition	$t(S, S') \equiv G'(v) = G(v) \oplus (G(u) \otimes E[u, v])$
Safety	$\forall S, S': t(S, S') \Rightarrow G'(v) \leq G(v)$
Progress	$\lvert\{E[u,v] \mid \text{untraversed}\}\rvert$

Visual Transition



Proof Refinement

∘({P}, ⊕, ⊗, 0, 1) → T

traverse(S, E[u,v], ∘) → G

idempotent semirings

reachability ({0, 1}, ∨, ∧, 0, 1)

shortest path ({ℝ⁺}, min, +, 0, ∞⁺)

Object Refinement

Semiring

GraphTraverser

Reachability — ShortestPath

Acknowledgements

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References

K. Mani Chandy, Brian Go, Sayan Mitra, and Jerome White. "Towards Verified Distributed Software Through Refinement of Formal Archetypes." Second IFIP Working Conference on Verified Software: Workshop on Experiments. October 6-9, 2008.

K. Mani Chandy, Brian Go, Sayan Mitra, Concetta Pilotto, and Jerome White. "Guaranteeing Convergence of Distributed Systems: From Specification to Implementation via Refinenement." Submitted: Formal Aspects of Computing.