Heavy-traffic Analysis of Mean Response Time Under SRPT

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Overview

Shortest Remaining Processing Time (SRPT) has long been known to optimize the queue length distribution and the mean response time. As such, it has been the focus of a wide body of analysis. But most results are very complex. For example, in an M/GI/1:

$$E[T]^{SRPT} = \int_0^1 \left(\int_0^x \frac{dt}{1-\rho(t)} + \frac{\lambda \int_0^x t\bar{F}(t)dt}{(1-\rho(x))^2} \right) dF(x) \quad \text{where} \quad \rho(x) = \lambda \int_0^x tf(t)dt$$

Numerical evaluation is harder than simulation.

Goal

Simple approximation for SRPT

Approach

In order to get a simpler formula:

- We look at heavy-traffic regime.
- Approximate SRPT by Preemptive Shortest Job First (PSJF).

$$E[T]^{SRPT} \le E[T]^{PSJF} \le \tfrac{3}{2} E[T]^{SRPT}$$

- Change the measure:

$$G(x) = \rho(x)/\rho = \int_0^x tf(x)/E[X]$$

- Assume $\lim_{x\to\infty} xh_F(x)$ exists(finite or infinite), $h_F(x) = f(x)/\bar{F}(x)$.

Results

Theorem. *For $M/GI/1$, the mean response time as $\rho \to 1$ is*

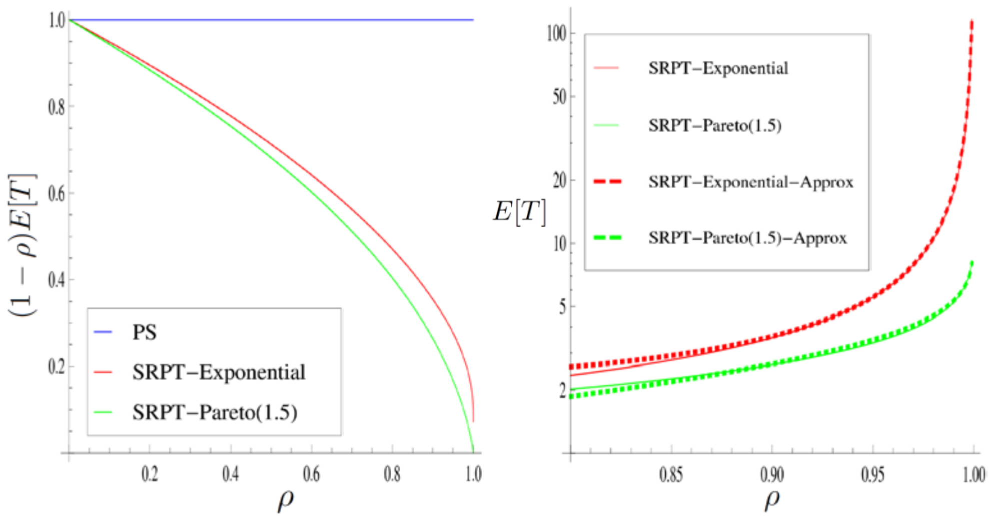
$$E[T]^{SRPT} = \begin{cases} \Theta\left(\log \frac{1}{1-\rho}\right) & F(x) \text{ has unbounded support and } 1 < \lim_{x\to\infty} xh_F(x) < 2 \\ \Theta\left(\frac{1}{(1-\rho)G^{-1}(\rho)}\right) & F(x) \text{ has unbounded support and } \lim_{x\to\infty} xh_F(x) > 2 \\ \Theta\left(\frac{1}{1-\rho}\right) & F(x) \text{ has bounded support} \end{cases}$$

Remarks:

1) Heavy-traffic behavior depends only on the tail of the job sizes.

2) SPRT has better performance when the job size tail is heavier.

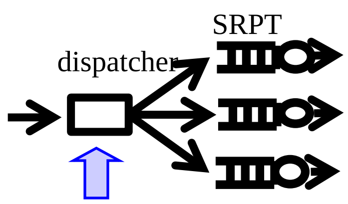
3) SRPT has much better performance than PS/FCFS, which have $E[T] = \Theta\left(\frac{1}{1-\rho}\right)$.



Applications

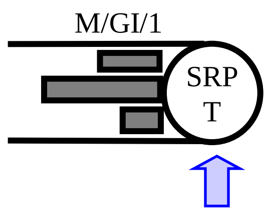
Having this simple approximation allows the study of SRPT in more complex models. Currently we are working on two applications.

Load Balancing Design



How should jobs be dispatched?

Power Management



What speed best balances Energy & Delay?

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