On the Capacity of Bounded Rank Modulation for Flash Memories

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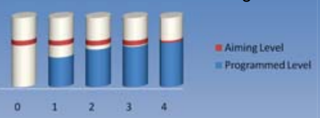
Background

•Flash Memory

✓Non-volatile storage using floating gate

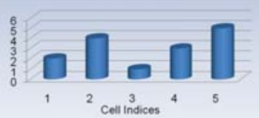
✓Block erasure & iterative writing



•Rank Modulation

✓Writing speed

✓Data reliability



Induced Permutation: (2,4,1,3,5)

✓Permutation size, m(=5);

✓Maximum cell level, D(=6), D≥m;

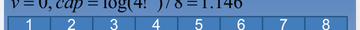
✓Overlap, v(=0), 0≤v≤m-1;

✓Capacity, cap(=log5!/5)

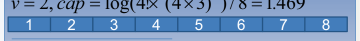
Bounded Rank Modulation (BRM)

•An example with 8 cells (m=4)

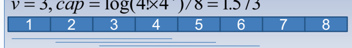
$v=0, cap=\log(4!^2)/8=1.146$



$v=2, cap=\log(4!\times(4\times3)^2)/8=1.469$



$v=3, cap=\log(4!\times4^4)/8=1.573$



•BRM code:

✓Bounded permutation size, m

✓Bounded maximum cell level, D

BRM Code with One Overlap I (Consecutive Levels)

•Method: Labeled graph

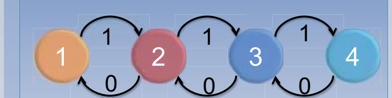
✓State: level of the current cell

✓Edge: possible cell-level transition

✓Labeling: induced permutation

✓Each block forms a set of m consecutive numbers

•m=2, D=4, v=1 (1="12", 0="21")



✓Adjacency matrix

$$A=\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&1&0&1\\0&0&1&0\end{bmatrix}$$

✓Capcity : $cap=\dfrac{\log\lambda(A)}{m-v}=.6942$

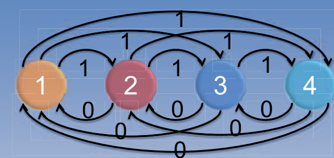
where $\lambda(A)$ is the largest positive eigenvalue of A

•Theorem: for m≥2 and D≥m+2, cap(m,D,v=1)>cap(m,D,v=0)

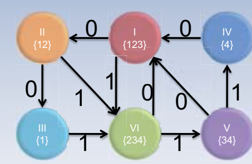
BRM Code with One Overlap II

•Each block is an arbitrary set of m distinct numbers

•m=2, D=4, v=1



•Deterministic representation

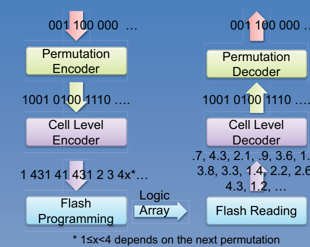


$$A=\begin{bmatrix}0&1&0&0&0&1\\0&0&1&0&0&1\\0&0&0&0&0&1\\1&0&0&0&0&0\\1&0&0&1&0&0\\1&0&0&0&1&0\end{bmatrix}$$

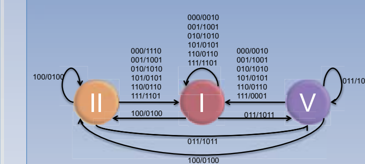
$cap=0.8791>0.6942$

Encoder and Decoder

•Diagram : Encoder/decoder (m=2, D=4, v=1)



✓3-state permutation encoder (rate=3:4). Labeling: information/ permutation



✓State- independent permutation decoder

First Two Permutation Bits	Permutation Sequence	Information Sequence
Equal	0010	000
	1110	000
	0001	111
	1101	111
Not equal	1011	011
	0100	100
	0101	101
	0110	110

✓ Cell level encoder

$$c_1=\begin{cases}1, & \text{if } P_0=1\\ 4, & \text{if } P_0=0\end{cases}$$

$$c_{i+1}=\begin{cases}c_i+1 & \text{if } P_{i-1}=1 \text{ and } P_i=1\\ 1 & \text{if } P_{i-1}=0 \text{ and } P_i=1\\ 4 & \text{if } P_{i-1}=1 \text{ and } P_i=0\\ c_i-1 & \text{if } P_{i-1}=0 \text{ and } P_i=0\end{cases}$$

✓Cell level decoder: 2-way comparator

✓Flash Programming: finding the maximum decreasing runs and writing from the lowest level

✓Flash Reading: sequentially reading off charge levels

Concluding Remarks

•Optimal overlap in terms of highest capacity for fixed D and m

✓When D=m, v*=0 optimizes capacity

✓When D=∞, v*=m-1 optimizes capacity

✓When D≥m+2,optimal v* ≥1

•Open problems

✓Exact optimal overlap for m<D<∞

✓Efficient encoder/decoder for arbitrary BRM codes

✓Generalized BRM code: other forms of overlap