Background

- **Flash Memory**
  - Non-volatile storage using floating gate
  - Block erasure & iterative writing

- **Rank Modulation**
  - Writing speed
  - Data reliability

- **Permutation size, m (=5);**
- **Maximum cell level, D (=6), D_m;**
- **Overlap, v (=0), 0 ≤ v ≤ m-1;**
- **Capacity, cap (=log_5 (m) / 5)**

Bounded Rank Modulation (BRM)

- **An example with 8 cells (m=4)**
  - v = 0, cap_v = log_5 (4^0) / 8 = 1.146
  - v = 2, cap_v = log_5 (4^2) / 8 = 1.469
  - v = 3, cap_v = log_5 (4^3) / 8 = 1.573

- **BRM code:**
  - Bounded permutation size, m
  - Bounded maximum cell level, D

BRM Code with One Overlap I (Consecutive Levels)

- **Method:** Labeled graph
- **State:** level of the current cell
- **Edge:** possible cell-level transition
- **Labeling:** induced permutation
- **Each block forms a set of m consecutive numbers**
- **m=2, D=4, v=1 (1=“12”, 0=“21”)**

- **Adjacency matrix**

\[ A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 \\
\end{bmatrix} \]

- **Capacity:**

\[ \text{cap} = \log_5 \lambda_1(A) = 0.6942 \]

- **Theorem:** for m ≥ 2 and D ≥ m, cap(m,D,v=1) > cap(m,D,v=0)

BRM Code with One Overlap II

- **Each block is an arbitrary set of m distinct numbers**
- **m=2, D=4, v=1**

- **Deterministic representation**

Encoder and Decoder

- **Diagram:** Encoder/decoder
  - m=2, D=4, v=1

- **3-state permutation encoder (rate=3:4).**
  - Labeling: information permutation

Concluding Remarks

- **Optimal overlap in terms of highest capacity for fixed D and m**
  - When D=m, v*=0 optimizes capacity
  - When D< m, v*=m-1 optimizes capacity
  - When D>m, optimal v* ≥ 1

- **Open problems**
  - Exact optimal overlap for m ≤ D ≤ c_m
  - Efficient encoder/decoder for arbitrary BRM codes
  - Generalized BRM code: other forms of overlap

- **Cell level encoder**
  - c_v = \begin{cases} 
  1, & \text{if } P_v = 1 \\
  4, & \text{if } P_v = 0 \\
  \end{cases}

- **Cell level decoder:** 2-way comparator
  - Flash Programming: finding the maximum decreasing runs and writing from the lowest level
  - Flash Reading: sequentially reading off charge levels