

Two-sided matching markets

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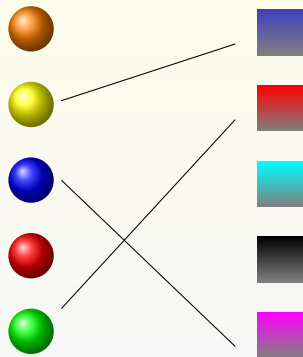
California Institute of Technology

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Two-sided market

Features:

- ▶ Indivisible (discrete) goods.
- ▶ Two sides



A match 

Examples

- ▶ Buyers and sellers.
- ▶ Workers and firms.
- ▶ Agents and objects (houses, human organs, ...).
- ▶ Auctions.
- ▶ Marriage.

Best known applications

- ▶ Hospitals and Interns (NRMP)
- ▶ Public Schools (NYC)

Marriage metaphor:

- ▶ a set M of *men*
- ▶ a set W of *women*
- ▶ each agent has *preferences* over members of opposite group

Example

$$W = \{w_1, w_2, w_3\}, M = \{m_1, m_2, m_3\},$$

$$P(w_1) : m_1, m_2$$

$$P(w_2) : m_1, m_3, m_2$$

$$P(w_3) : m_2, m_3, m_1$$

$$P(m_1) : w_3, w_2, w_1$$

$$P(m_2) : w_1, w_2$$

$$P(m_3) : w_2, w_1, w_3$$

Example

$P(w_1) : m_1, m_2$

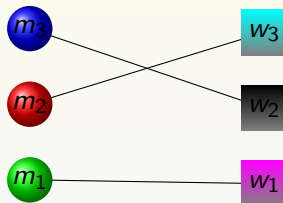
$P(w_2) : m_1, m_3, m_2$

$P(w_3) : m_2, m_3, m_1$

$P(m_1) : w_3, w_2, w_1$

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$P(m_3) : w_2, w_1, w_3$



A *matching* μ describes who is matched with whom.

- ▶ $\mu(m_2) = w_3$ means m_2 and w_3 are matched.
- ▶ $\mu(m_2) = m_2$ means that m_2 is single.

A matching is *individually rational* if no agent who is matched is better off being single.

Individual Rationality

A matching is *individually rational* if no agent who is matched is better off being single.

A pair (m, w) block a matching μ if they are not matched in μ and

- ▶ m prefers w over $\mu(m)$
- ▶ w prefers m over $\mu(w)$

A matching is *stable* if it is individually rational and no pair can block it.

Stable matchings exist.

In fact, there are two distinguished matchings μ_M and μ_W s.t. All men prefer μ_M to any other stable matching, and prefer any stable matching to μ_W ; and vice versa.

- ▶ μ_M is *men optimal* and *women pessimal*
- ▶ μ_W is *women optimal* and *men pessimal*

So there is *coincidence of interest* where you might not expect it.
Why ?

Example

$$P(w_1) : m_1, m_2$$

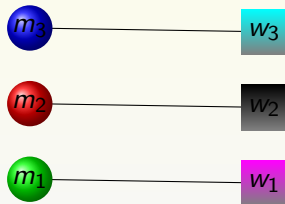
$$P(w_2) : m_1, m_3, m_2$$

$$P(w_3) : m_2, m_3, m_1$$

$$P(m_1) : w_3, w_2, w_1$$

$$P(m_2) : w_1, w_2$$

$$P(m_3) : w_2, w_1, w_3$$



(m_1, w_2) block

Example

$P(w_1) : m_1, m_2$

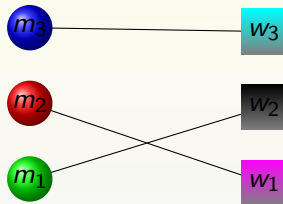
$P(w_2) : m_1, m_3, m_2$

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$P(m_1) : w_3, w_2, w_1$

$P(m_2) : w_1, w_2$

$P(m_3) : w_2, w_1, w_3$



women optimal

Example

$P(w_1) : m_1, m_2$

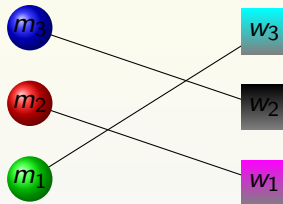
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$P(m_1) : w_3, w_2, w_1$

$P(m_2) : w_1, w_2$

$P(m_3) : w_2, w_1, w_3$



men optimal

Purpose:

- ▶ Test tendency of mkts. to stabilize
- ▶ Which stable matching is selected?

Joint work with Gabriel Katz and Leeat Yariv.

Divide subjects into two groups (colors and foods); present them with payoffs from each match.

Allow anyone to propose a match at any time.

Experimental Interface

Subject ID: 13

My Type: cyan

Time Left: 9 sec

	apple	banana	kiwi	cherry	mango	pear	grape	peach
red	10, 1	20, 2	30, 3	40, 4	50, 5	60, 6	70, 7	80, 8
blue	90, 9	100, 10	110, 11	120, 12	130, 13	140, 14	150, 15	160, 16
green	170, 17	180, 18	190, 19	200, 20	210, 21	220, 22	230, 23	240, 24
magenta	250, 25	260, 26	270, 27	280, 28	290, 29	300, 30	310, 31	320, 32
yellow	330, 33	340, 34	350, 35	360, 36	370, 37	380, 38	390, 39	400, 40
pink	410, 41	420, 42	430, 43	440, 44	450, 45	460, 46	470, 47	480, 48
cyan	490, 49	500, 50	510, 51	520, 52	530, 53	540, 54	550, 55	560, 56
orange	570, 57	580, 58	590, 59	600, 60	610, 61	620, 62	630, 63	640, 64

	apple	banana	kiwi	cherry	mango	pear	grape	peach
red								
blue								
green								
magenta								
yellow								
pink								
cyan								
orange								

Your History

Round 1

Practice Round [Switch to Full View](#)

Round Payoff: 0 (\$0.00)

Type	Matched With	My Role	Payoff
cyan			

Payoffs

330,310	540,140	190,190	400,350	50,240	120,240	470,330	260,540
240,100	380,420	100,50	310,70	520,380	450,450	170,190	30,330
240,240	170,560	520,260	310,140	30,520	380,520	100,540	450,400
350,450	210,210	0,330	420,280	140,170	70,170	280,470	490,120
190,170	120,490	330,540	260,490	540,310	50,380	400,400	470,260
210,520	490,280	420,120	560,210	350,450	70,590	280,260	140,190
240,30	170,70	450,400	310,0	380,590	520,310	100,120	30,470
490,380	350,350	140,470	280,420	70,660	420,100	210,50	560,50

Payoffs

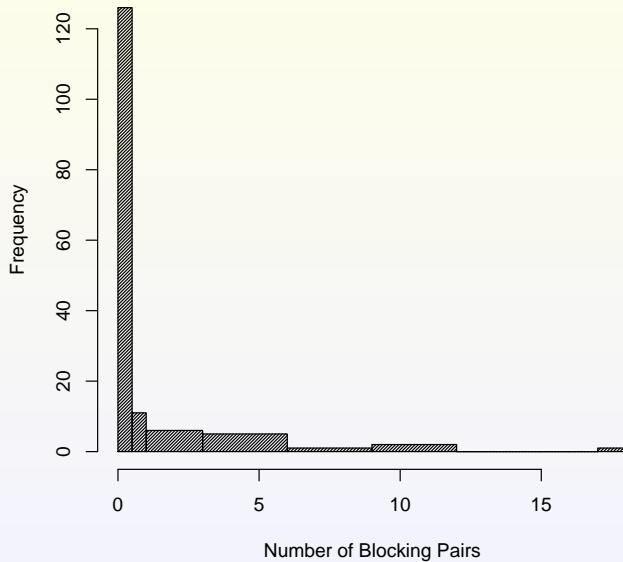
300,790	370,860	650,90	230,650	510,860	930,90	580,300	20,440	720,790	860,580	440,20	160,650	1000,160	90,580
650,230	20,650	790,440	860,440	720,160	230,650	90,650	300,90	1000,90	440,370	930,510	580,720	160,650	510,930
90,510	580,160	860,510	370,930	160,790	440,580	790,20	510,930	930,370	230,860	650,370	300,930	20,230	720,790
160,930	790,790	20,370	860,370	510,510	650,300	300,860	930,510	440,1000	580,930	90,650	370,510	230,860	720,230
930,370	90,580	510,860	580,230	300,930	720,790	20,580	650,230	230,720	370,1000	860,580	790,300	1000,20	160,650
720,20	790,20	160,1000	510,300	440,300	370,930	230,720	1000,160	930,300	650,90	580,790	20,1000	860,510	90,1000
230,650	580,440	720,720	370,510	160,650	930,510	300,1000	650,300	1000,440	790,20	20,230	860,370	440,1000	510,860
580,720	860,90	370,790	440,860	930,440	90,440	20,440	160,650	650,230	230,650	510,300	720,20	1000,370	300,790
160,1000	580,930	20,300	440,790	90,1000	650,230	860,370	790,79	510,580	1000,510	720,440	300,790	230,580	930,510
1000,90	580,230	160,650	90,90	790,230	300,860	650,790	230,720	20,20	720,160	370,860	510,160	440,930	860,440
720,300	860,510	650,160	790,20	1000,20	230,720	440,230	300,580	370,930	580,790	930,160	90,580	160,720	510,300
790,160	650,300	90,580	230,580	300,90	440,20	930,510	370,860	160,860	1000,440	510,930	580,230	720,790	860,300
510,860	300,720	650,230	160,1000	720,720	370,1000	90,90	860,370	930,510	230,720	440,1000	790,90	580,440	20,230
160,580	370,1000	860,20	230,160	510,370	1000,370	440,930	300,1000	790,160	720,230	20,720	930,440	90,90	580,160
930,440	90,370	370,930	720,720	300,580	790,160	160,160	860,20	230,650	580,300	650,90	510,860	1000,300	440,930

Main findings

- ▶ Stability is a very good predictor of market outcomes.
- ▶ Markets select a median matching, when one exists.
- ▶ The “hardest” markets for the agents were not the most complex but the one where competition on the same side was the harshest.

		Stable	O. F.	O. C.	Median	Unstable	Mkt. Stable
8×8	Unique	95%				5%	92%
	Two	94%	61%	39%		6%	91%
	Three	95%	11%	16%	73%	5%	72%
	Total	95%				5%	85%
15×15	Unique	83%				17%	50%
	Three	100%	3%	3%	93%	0%	100%
	Total	89%				11%	80%

Histogram Blocking Pairs



	Stable	O. Food	O. Color	Median	Unstable
Unique	83%				17%
Two	97%	92%	8%		3%
Three	81%	29%	12%	59%	19%
Total	87%				13%

Table: Unilateral treatment: Only Foods propose.